GRADE 12 (MHF4U1)

SECONDARY SCHOOL MATHEMATICS TESTS AND EXAMS

(WITH COMPLETE SOLUTIONS)

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CONTENTS

TEST #3

1. Divide. Put your answers in the form $f(x) = d(x)g(x) + r(x)$ and state any restrictions on the variable.

   a) $(x^5 - x^3 - x + 2) \div (x^2 + 3)$

   b) $(2x^3 - 7x^2 + 9x - 3) \div (2x - 1)$

2. If $x^4 + x^3 + ax^2 + bx - 3$ is divisible by $x - 1$ and $x - 3$, find the values of a and b.

3. Factor completely:

   a) $x^3 - 3x^2 - x + 3$

   b) $2x^3 + 5 - 4$

   c) $2x^3 - 5x^2 - 4x + 3$

   d) $x^4 - x^3 - 7x^2 + x + 6$

4. a) Determine the equation of a cubic function whose x-intercepts are 3, 1, and -2 and y-intercept is 12.

   b) Determine the equation of a quartic function with a root at $x = 2$, triple root at 5, and passes through (7, 10).

5. Describe the transformations needed to transform $y = x^3$ to $y = -3(2x - 4)^3 + 5$.

6. Find the remainder when $x^4 - 3x^3 + 4x^2 - 3x + 5$ is divided by $x + 5$.

7. Sketch a possible graph of $y = 2x(x + 3)^2(x - 5)^3$.

8. Describe the end behaviour of the following:

   a) $y = -3x^3 + 5x + 7$

   b) $y = 5 - x^4 - x^2 + 6$

9. State the degree and x-intercepts of the following polynomial functions and draw a sketch of each graph.

   a) $f(x) = -(x - 2)^2(x + 1)$

   b) $f(x) = x(x + 3)^2(x + 1)(2x - 5)$

TEST 3 Solutions

1. a)

$$x^2+3 \;\overline{)\; x^5+0x^4-x^3+0x^2-x+2}$$

$$\begin{array}{r} x^3 \quad -4x \\ \hline x^5+0x^4-x^3+0x^2-x+2 \\ x^5 \quad +3x^3 \\ \hline -4x^3+0x^2-x \\ -4x^3 \quad -12x \\ \hline 11x+2 \end{array}$$

There are no restrictions since x^2+3 can't equal zero.

$$x^5-x^3-x+2 = (x^2+3)(x^3-4x)+11x+2$$

b)

$$2x-1 \;\overline{)\; 2x^3-7x^2+9x-3}$$

$$\begin{array}{r} x^2-3x+3 \\ \hline 2x^3-7x^2+9x-3 \\ 2x^3-1x^2 \\ \hline -6x^2+9x \\ -6x^2+3x \\ \hline 6x-3 \\ 6x-3 \\ \hline 0 \end{array}$$

Restriction:

$2x-1 \neq 0$

$2x \neq 1$

$x \neq \frac{1}{2}$

$$2x^3-7x^2+9x-3 = (2x-1)(x^2-3x+3)$$

2. Let $f(x) = x^4+x^3+ax^2+bx-3$

If $f(x)$ is divisible by $x-1$,

$f(1) = 0$

$f(1) = (1)^4+(1)^3+a(1)^2+b(1)-3$

$0 = a+b-1$

① $1 = a+b$

If $f(x)$ is divisible by $x-3$,

$f(3) = 0$

$f(3) = (3)^4+(3)^3+a(3)^2+b(3)-3$

$0 = 9a+3b+105$

$-105 = 9a+3b$ ÷3

② $-35 = 3a+b$

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TEST 3 Solutions

2. (Continued)

$$\begin{aligned} &② \quad -35 = 3a + b \\ &① \quad 1 = a + b \\ \hline &② - ① \quad -36 = 2a \\ &\qquad -18 = a \end{aligned}$$

→ Let $a = -18$ in ①

① $1 = -18 + b$

$19 = b$

$\therefore a = -18$ and $b = 19$

3. a) $x^3 - 3x^2 - x + 3$

$= x^2(x-3) - 1(x-3)$

$= (x-3)(x^2-1)$

$= (x-3)(x+1)(x-1)$

b) $2x^3 + 54$

$= 2[x^3 + 27]$

$= 2(x+3)(x^2 - 3x + 9)$ ⤵ *

* $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

c) $2x^3 - 5x^2 - 4x + 3$

Possible zeros are

$x = \pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$

Try $x = 1$, $2 - 5 - 4 + 3 \neq 0$

Try $x = -1$, $-2 - 5 + 4 + 3 = 0$ $\Longrightarrow$ $x+1$ is a factor

$$\begin{array}{r} 2x^2 - 7x + 3 \\ x+1 \overline{)\,2x^3 - 5x^2 - 4x + 3} \\ \underline{2x^3 + 2x^2} \\ -7x^2 - 4x \\ \underline{-7x^2 - 7x} \\ 3x + 3 \\ \underline{3x + 3} \\ 0 \end{array}$$

→ $2x^3 - 5x^2 - 4x + 3$

$= (x+1)(2x^2 - 7x + 3)$

$= (x+1)(2x-1)(x-3)$